

## CS3221 Exam 2, Spring 2026

Your name:

**Q1:** For what sort of graph would an adjacency matrix be more space-efficient than an adjacency list? Why? Be specific.

**Q2:** If you do a DFS search and classify edges on a graph with 12 vertices and 15 edges, how many edges will be tree edges? Prove it.

**Q3:** Prove that every tree with directed edges directed from root to leaves has a topological ordering.

**Q4:** Given a connected, weighted, undirected graph  $G$ , prove that for any cycle in  $G$ , the minimum spanning tree of  $G$  excludes the maximum-weight edge in that cycle.

**Q5:** You are given a connected, undirected graph  $G$  with weighted edges and a minimum spanning tree  $T$  of  $G$ . The weight of a single edge  $e$  is decreased. Describe an algorithm to update the minimum spanning tree efficiently.

**Q6:** Describe and analyze an algorithm to compute the maximum-weight spanning tree of a given edge-weighted graph  $G$ .

**Q7:** You are given a directed graph with edge weights that may be negative. Describe an algorithm to determine whether there exists a path from  $s$  to  $t$  with total weight exactly equal to  $k$  (for some given  $k$ ), assuming you only need to answer yes/no.

**Q8:** Given a flow network where every edge has capacity 1, what is the relationship between the value of the maximum flow and the number of edge-disjoint paths from  $s$  to  $t$ ?